CRD IV

4 May 2012

Credit Valuation Adjustment (CVA) - Advanced CVA formula

The Issue

The advanced CVA formula as currently written in the Basel III document can be treated as the following, broadly the integral over time of the LGD * PD * Exposure:

$$CVA = \underbrace{LGD_{MKT}}_{\text{<<LGD>>}} \cdot \sum_{i=1}^{T} \underbrace{\max\left\{0, \exp\left(-\frac{s_{i-1} \cdot t_{i-1}}{LGD_{MKT}}\right) - \exp\left(-\frac{s_i \cdot t_i}{LGD_{MKT}}\right)\right\}}_{\text{<<Default probability occurring in the period>>}} \cdot \underbrace{\frac{EE_{i-1} \cdot D_{i-1} + EE_i \cdot D_i}{2}}_{\text{<< Average exposure in the period>>}}$$

However, the first term of this formula, which is the Loss Given Default for the perspective instrument, is using the Market LGD. Therefore, the formula does not allow flexibility in using a netting set specific LGD to reflect the seniority compared to those derivatives traded in the market.

This was explained in the "Basel III counterparty credit risk – Frequently asked questions" issued by the BCBS in November 2011 ("the FAQ"), where the BCBS stated that:

> "LGD_{MKT} needs to be consistent with the derivation of the hazard rates – and therefore must reflect market expectations of recovery rather than mitigants or experience specific to the firm."

However, in response to question 22 of the FAQ, the BCBS permit the following alternative for the LGD term:

> "In cases where a netting set of derivatives has a different seniority than those derivative instruments that trade in the market from which LGD_{MKT} is inferred, a bank may adjust LGD_{MKT} to reflect this difference in seniority."

Moreover, the CVA formula in the current CRD IV package contains a typo where the last term should be the average exposure.

Changes proposed to Article 373

In the circumstances described in the BCBS's response to question 22, it would seem appropriate to adjust the CVA formula from CRD IV as follows:

$$CVA = LGD_{NS} \cdot \sum_{i=1}^{T} \max\left\{0, \exp\left(-\frac{s_{i-1} \cdot t_{i-1}}{LGD_{MKT}}\right) - \exp\left(-\frac{s_i \cdot t_i}{LGD_{MKT}}\right)\right\} \cdot \frac{EE_{i-1} \cdot D_{i-1} + EE_i \cdot D_i}{2}$$

Where LGD_{NS} is the adjusted LGD used to reflect the netting set of the derivative.

This formulation preserves the correct hazard rates (represented by the middle term of the formula) as required by the BCBS's response to question 21 of the FAQ but allows the CVA to be scaled appropriately for the netting set of the instruments in question. Thus, the probability of the counterparty defaulting remains unaffected but the loss resulting from the derivatives of a different seniority is adjusted in line with that seniority.

In addition, the current Commission proposed CRD IV package includes a typo in the final term of the advanced CVA formula, where it should be a plus sign instead of minus sign.

Under this formulation of CVA the Regulatory CS01 formulae (for specific tenor and parallel shift credit spread sensitivities respectively) would need to be recast as follows:

$$CS01_i = 0.0001 \cdot \frac{LGD_{NS}}{LGD_{MKT}} \cdot t_i \cdot \exp\left(-\frac{s_i \cdot t_i}{LGD_{MKT}}\right) \cdot \left(\frac{EE_{i-1} \cdot D_{i-1} - EE_{i+1} \cdot D_{i+1}}{2}\right)$$

A detailed derivation can be found in Appendix of this paper.

For the final time bucket i=T, the corresponding formula is:

$$\text{Regulatory}CS01_T = 0.0001 \cdot \frac{LGD_{NS}}{LGD_{MKT}} \bullet t_T \cdot \exp\left(\frac{s_T \bullet t_T}{LGD_{MKT}}\right) \bullet \frac{EE_{T-1} \cdot D_{T-1} + EE_T \cdot D_T}{2}$$

$$CS01 = 0.0001 \cdot \frac{LGD_{NS}}{LGD_{MKT}} \cdot \sum_{i=1}^{T}\left(t_i \cdot \exp\left(-\frac{s_i \cdot t_i}{LGD_{MKT}}\right) - t_{i-1} \cdot \exp\left(-\frac{s_{i-1} \cdot t_{i-1}}{LGD_{MKT}}\right)\right) \cdot \left(\frac{EE_{i-1} \cdot D_{i-1} + EE_i \cdot D_i}{2}\right)$$

Proposed changes

The current Commission proposed CRD IV package includes a typo in the final term in CRR Article 373 and should be corrected.

$$\cancel{CVA = LGD_{MKT} \cdot \sum_{i=1}^{T} \max\left\{0, \exp\left(\frac{s_{i-1} \cdot t_{i-1}}{LGD_{MKT}}\right) - \exp\left(\frac{s_i \cdot t_i}{LGD_{MKT}}\right)\right\} \cdot \frac{EE_{i-1} \cdot D_{i-1} - EE_i \cdot D_i}{2}}$$

$$CVA = LGD_{MKT} \cdot \sum_{i=1}^{T} \max\left\{0, \exp\left(-\frac{s_{i-1} \cdot t_{i-1}}{LGD_{MKT}}\right) - \exp\left(-\frac{s_i \cdot t_i}{LGD_{MKT}}\right)\right\} \cdot \frac{EE_{i-1} \cdot D_{i-1} + EE_i \cdot D_i}{2}$$

Internal Model Approach: Institutions which are using internal models for debt instruments may not have the CVA calculation integrated into their systems. Therefore, flexibility should be given for institutions to use the standardised approach if they have not developed the advanced method. In addition, compared to Basel III, the multiplier in CRR Article 373 is not clearly defined. AFME has proposed changes to CRR Article 373 to deal with these issues.

Appendix

CVA Formula to CS01 Formula where t<T and the model is based on credit spread sensitivities for specific tenors

$$CVA = LGD_{NS} \cdot \sum_{i=1}^{T} \max\left\{0, \exp\left(-\frac{s_{i-1} \cdot t_{i-1}}{LGD_{MKT}}\right) - \exp\left(-\frac{s_i \cdot t_i}{LGD_{MKT}}\right)\right\} \cdot \frac{EE_{i-1} \cdot D_{i-1} + EE_i \cdot D_i}{2}$$

Collecting s_i terms we have:

$$CVA = LGD_{NS} \cdot \sum_{i=1}^{T-1} \exp\left(-\frac{s_i \cdot t_i}{LGD_{MKT}}\right) \cdot \frac{EE_{i+1} \cdot D_{i+1} - EE_{i-1} \cdot D_{i-1}}{2} + C_T + C_0$$

Where

$$\sum_{i=1}^{T} \left\{\exp\left(-\frac{s_{i-1} \cdot t_{i-1}}{LGD_{MKT}}\right)\right\} \cdot \frac{EE_{i-1} \cdot D_{i-1} + EE_i \cdot D_i}{2}$$

$$= \sum_{i=0}^{T-1} \left\{\exp\left(\frac{-s_{i-1} \cdot t_{i-1}}{LGD_{MKT}}\right)\right\} \cdot \frac{EE_{i-1} \cdot D_{i-1} + EE_i \cdot D_i}{2}$$

$$= \sum_{i=1}^{T-1} \left\{\exp\left(\frac{-s_{i-1} \cdot t_{i-1}}{LGD_{MKT}}\right)\right\} \cdot \frac{EE_{i-1} \cdot D_{i-1} + EE_i \cdot D_i}{2} + \underbrace{\exp\left(\frac{S_0 T_0}{LGD_{MKT}}\right) \cdot \frac{EE_{i-1} \cdot D_{i-1} + EE_i \cdot D_i}{2}}_{\dfrac{C_0}{LGD_{NS}}}$$

and

$$-\sum_{i=1}^{T} \left\{\exp\left(-\frac{s_i \cdot t_i}{LGD_{MKT}}\right)\right\} \cdot \frac{EE_{i-1} \cdot D_{i-1} + EE_i \cdot D_i}{2}$$

$$= -\sum_{i=0}^{T-1} \left\{\exp\left(-\frac{s_i \cdot t_i}{LGD_{MKT}}\right)\right\} \cdot \frac{EE_{i-1} \cdot D_{i-1} + EE_i \cdot D_i}{2} - \underbrace{\exp\left(\frac{-S_T t_T}{LGD_{MKT}}\right) \cdot \frac{EE_{T-1} D_{T-1} + EE_T D_T}{2}}_{\dfrac{C_T}{LGD_{NS}}}$$

and

$$\sum_{i=1}^{T-1} \left\{ \exp\left(\frac{-s_{i-1} \cdot t_{i-1}}{LGD_{MKT}} \right) \right\} \cdot \left[\frac{EE_i \cdot D_i + EE_{i+1} \cdot D_{i+1}}{2} - \frac{EE_{i-1} \cdot D_{i-1} + EE_i \cdot D_i}{2} \right]$$

$$= \sum_{i=1}^{T-1} \left\{ \exp\left(\frac{-s_{i-1} \cdot t_{i-1}}{LGD_{MKT}} \right) \right\} \cdot \left[\frac{EE_{i+1} \cdot D_{i+1} - EE_{i-1} \cdot D_{i-1}}{2} \right]$$

Therefore

$$CS01_i = 0.0001 \cdot \frac{LGD_{NS}}{LGD_{MKT}} \cdot t_i \cdot \exp\left(-\frac{s_i \cdot t_i}{LGD_{MKT}} \right) \cdot \left(\frac{EE_{i-1} \cdot D_{i-1} - EE_{i+1} \cdot D_{i+1}}{2} \right)$$

(Differentiating the exponential function (of $-s_i.t_i/LGD_{mkt}$) gives a resultant equation where $-t_i/LGD_{mkt}$ is a factor. The minus sign in this factor swaps the $EE_{i+1}D_{i+1}$ and $EE_{i-1}D_{i-1}$ terms.)